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A Modified Parton Model for the Deep Inelastic Form Factors of the Nucleon

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ABSTRACT

We construct a three quark parton model where the final parton is allowed to have a mass different from the initial one. The structure functions become convolutions of the momentum and mass distributions. The model gives good fits to the data for νW_2, it has a fast but not instantaneous approach to scaling and satisfies the sum rule. It leads to a finite W_1 and to a constant ratio for F_n/F_p in contradiction with experiment.

I. INTRODUCTION

We study the properties of a modified parton[1] model which we develop from the following assumptions:

a) The nucleon (initial state) contains exactly three partons or quarks.

b) The final state contains three quarks plus an arbitrary number of quark-antiquark pairs in the form of mesons.

c) The mass of the final (intermediate) parton is allowed to be different from the one in the initial state.

The picture we have in mind is shown in Fig. 1. A virtual photon is absorbed by one of the three quarks which make up the nucleon. The nucleon then fragments into a baryon and many mesons. The active quark that absorbed the initial photon enters any one of the final particles.

The construction of such a model is of interest for several reasons. The mass of the parton in a final particle has to be determined by that particle and will in general be different from the one in the proton. The quark will therefore undergo a mass change in this model. The possible final mass will have some sort of statistical distribution depending on the final state parameters. The Bjorken-Paschos model[1] is the special case where this distribution is a delta-function.

A model in which the parton undergoes a discrete mass change has been considered by O. W. Greenberg and D. Bhaumik.[2] They show that the approach to scaling is affected by the mass of the intermediate state parton. Their curves for W_1 and νW_2 in the scaling limit[3] are the same as those of Bjorken-Paschos.[1] We show that both the approach to the scaling limit and the curves reached in the scaling limit depend on the parton mass distribution. The influence of other effects such as a distribution in the transverse momenta has been studied by Gardiner and Majumdar.[2]

In the Bjorken-Paschos model[1] the scaling limit is reached instanly and the curves for W_1 and νW_2 are proportional to the parton fractional momentum distribution f(x). Our model shows that this direct relationship can break down if the intermediate parton's mass varies. The fractional momentum distribution in our model must be convoluted with the intermediate mass distributions to get the structure functions and therefore the scaling limits.

The data[4] show that the scaling limit is reached quite fast; precosciuos scaling. Our model has a quite realistically fast, but not instantaneous, approach to scaling. Our model also shows that a three quark model can be modified such that it is possible to satisfy the Bjorken-Paschos[1] sum rule. This is done without the introduction of a meson cloud as was proposed by Kuti and Weisskopf.[5]

II. THE INELASTIC FORM FACTORS

In this section we give the definition of the deep inelastic structure functions, their relationship to virtual-photon nucleon scattering and derive the structure functions for an elementary photon parton process where the partons have different mass.

The total cross section for virtual-photon nucleon scattering is proportional to the tensor

$$W_{\mu\nu} = 4\pi^2 \frac{E_p}{m} \frac{1}{2} \sum_{s,n} \langle p,s| J_\mu(0)|n\rangle \langle n| J_\nu(0)|p,s\rangle \, (2\pi)^4 \delta^{(4)}(p+q-p_n) \tag{1}$$

$$= 4\pi^2 \frac{E_p}{m} \frac{1}{2} \sum_{s} \int d^4x \, e^{iqx} \langle p,s| J_\mu(x) J_\nu(0)|p,s\rangle . \tag{2}$$

Here $|p, s\rangle$ is a nucleon state with z-spin s, momentum p and normalized to

$$\langle p,s|p',s'\rangle = \delta^{(3)}(p-p')\,\delta_{s,s'} \tag{3}$$

Current conservation and Lorentz invariance imply that this tensor has the general form

$$W_{\mu\nu} = -\Delta_{\mu\nu}(q)\, W_1(\nu,q^2) + \frac{1}{m^2} \Delta_{\mu\alpha}(q) p^\alpha \Delta_{\nu\beta}(q) p^\beta\, W_2(\nu,q^2) \tag{4}$$

where

$$\Delta_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} , \tag{5}$$

and

$$\nu = p\cdot q/M \tag{6}$$

$$t = -q^2 . \tag{7}$$

Expression (4) defines the structure functions W_1 and W_2.

The T-matrix for forward $\gamma + p \to \gamma + p$ elastic scattering is, apart from the photon wave functions,

$$i\,T_{\mu\nu}(\nu, q^2)\,(2\pi)^4\,\delta(0) = \tag{8}$$

$$= \sum_s \frac{(-i)^2}{2} \int d^4x\, d^4y\, e^{iqx} \langle p, s | T(J_\mu(x+y) J_\nu(y)) | p, s \rangle,$$

and, by translation invariance,

$$T_{\mu\nu}(\nu, q^2) = -i \sum_s \frac{(-i)^2}{2} \int d^4x\, e^{iqx} \langle p, s | T(J_\mu(x) J_\nu(0)) | p, s \rangle, \tag{9}$$

The virtual γ-p cross section $W_{\mu\nu}$ is related to the imaginary part of the forward γ-p amplitude by (see Appendix I)

$$W_{\mu\nu} = 8\pi^2 \frac{E_p}{m} \,\mathrm{Im}\, T_{\mu\nu}. \tag{10}$$

We now consider the process shown in Fig. 2 where a virtual photon scatters elastically off a parton with spin 1/2 mass m_1 and momentum h, forming an intermediate parton of mass m_2. We can compute $T_{\mu\nu}$ for this process using the Feynman rules given in Bjorken and Drell.[6] We get

$$T_{\mu\nu} = \frac{-i}{2} \sum_s \frac{m_1}{h_0} \frac{1}{(2\pi)^3}\, \bar{u}^s\!\left(\frac{h}{m_1}\right)(-i\gamma_\mu) \frac{i}{\not h + \not q - m_2 + i\epsilon} (-i\gamma_\nu)\, u^s\!\left(\frac{h}{m_1}\right) \tag{11}$$

$$= \frac{-1}{2(2\pi)^3} \frac{m_1}{h_0} \mathrm{Tr}\left[\left(\frac{\not h + m_1}{2m_1}\right)\gamma_\mu (\not h + \not q + m_2)\gamma_\nu\right] \frac{1}{(h+q)^2 - m_2^2 + i\epsilon}. \tag{12}$$

Therefore, using the discontinuity formula (10) we find for the structure tensor $w_{\mu\nu}$ of this spin 1/2 parton

$$w_{\mu\nu} = \frac{1}{2} Tr\left[\frac{\not{h}+m}{2m_1}\gamma_\nu(\not{h}+\not{q}+m_2)\gamma_\mu\right]\delta((h+q)^2-m_2^2). \tag{13}$$

Note that the photon wave functions will be orthogonal to q. Therefore, we can contract $W_{\alpha\beta}$ with $\Delta_{\mu\alpha}\Delta_{\nu\beta}$ without loss of generality. The terms proportional to q drop out and we obtain

$$w_{\mu\nu}(h,m_1,m_2) = \left[-w_1\Delta_{\mu\nu}(q) + w_2\Delta_{\mu\alpha}(q)\frac{h_\alpha h_\beta}{m_1^2}\Delta_{\nu\beta}(q)\right], \tag{14}$$

with

$$w_1 = \frac{1}{m_1}\left[m_1^2 + h\cdot q - m_1 m_2\right]\delta((k+q)^2-m_2^2) \tag{15}$$

$$w_2 = 2m_1\,\delta((h+q)^2-m_2^2), \tag{16}$$

which, for $m_1 = m_2$, is in agreement with Bjorken-Paschos.[1]

III. THE STRUCTURE FUNCTIONS

The elastic scattering of a virtual photon on a three-quark nucleon is shown in Fig. 3. A quark of momentum h, mass $m_1 = \sqrt{h^2}$, and charge $g_i e$ (e = proton charge) absorbs a photon and enters a meson in the intermediate state with a mass m_2. Each such quark contributes to $w_{\mu\nu}$ a term $g_i^2\ w_{\mu\nu}(h, m_1, m_2)$. We assume that for large proton four-momentum p,

$$h = xp, \quad 0 \le x \le 1, \tag{17}$$

with probability f(x), independent of i. We also have a mass distribution g(y) such that

$$(m_2)^2 = y(p+q)^2 = ys, \quad 0 \leq y \leq 1. \tag{18}$$

Then the total structure tensor is the weighted integral over the individual parton tensors

$$W_{\mu\nu} = \int_0^1 dx \int_0^1 dy\, f(x) g(y)\, w_{\mu\nu}(xp, xM, \sqrt{ys}). \tag{19}$$

Therefore

$$W_1 = \int_0^1 dx\, dy\, f(x) g(y) [xM + \nu - \sqrt{ys}]\, \delta((xp+q)^2 - ys)(\Sigma g_i^2), \tag{20}$$

$$W_2 = 2M \int_0^1 dx\, dy\, f(x) g(y)\, x\, \delta((xp+q)^2 - ys)(\Sigma g_i^2). \tag{21}$$

In the usual three quark model one has

$$\sum_1^3 g_i^2 = 1, \text{ for the proton.} \tag{22}$$

$$\sum_1^3 g_i^2 = \frac{2}{3}, \text{ for the neutron} \tag{23}$$

In reality the vertex for $\gamma + m_1 \rightarrow m_2$ cannot be a simple γ_μ. There are extra terms which are required in order to satisfy the Ward identities. We are neglecting such corrections in the present work.

IV. THE BJORKEN LIMIT

We take the Bjorken limit where $t \to \infty$ with $w = 2M\nu/t$ fixed. Then $s = M^2 + t(w-1) \to t(w-1)$ and we find

$$\nu W_2(\nu, q^2) \to t\omega \int_0^1 dx\, dy\, x f(x) g(y)\, \delta[t(x\omega - 1) - y t(\omega - 1)] \tag{24}$$

$$= \frac{\omega}{\omega - 1} \int_{x_0}^1 dx\, x f(x)\, g\left(\frac{x\omega - 1}{\omega - 1}\right) \equiv F_2(\omega), \tag{25}$$

where x_o is determined by

$$0 \leq \frac{x\omega - 1}{\omega - 1} \leq 1, \tag{26}$$

which implies

$$\frac{1}{\omega} \leq x \leq 1. \tag{27}$$

Therefore

$$x_0 = 1/\omega. \tag{28}$$

In the same limit we find

$$2M W_1(\nu, q^2) \longrightarrow F_1(\omega), \tag{29}$$

where

$$F_1(\omega) = \frac{\omega}{\omega - 1} \int_{1/\omega}^1 dx\, f(x)\, g\left(\frac{x\omega - 1}{\omega - 1}\right). \tag{30}$$

V. THE DISTRIBUTION FUNCTIONS

We would expect the imtermediate state quark to have a mass m_1^2 near m_π^2 if it should enter a pion. Its mass squared will be a somewhat greater fraction of s if it ends up in a meson resonance. Only those final (intermediate) states which consist of single baryon resonances will give an appreciable probability that the quark mass will be near s. In the absence of any precise information, we will assume

$$g(y) \equiv 1. \tag{31}$$

Let $f(x_1, x_2, x_3)$ be the probability that the three nucleon quarks will divide up the nucleon momentum p in the fractions x_1, x_2, x_3 with

$$\sum_1^3 x_i = 1. \tag{32}$$

Then conservation of momentum implies

$$\prod_{i=1}^{3} \int d^3x_i \, \delta(1 - \sum_1^3 x_j) f(x_1, x_2, x_3) = 1. \tag{33}$$

We also have the single parton, distribution function f(x)

$$f(x) = \int_0^1 dx_2 \int_0^1 dx_3 \, \delta(1-x-x_2-x_3) f(x, x_2, x_3). \tag{34}$$

VI. LARGE w LIMITS

The assumption (31) leads to an interesting limit. From (25) we find sucessively

$$F_2(\infty) = \lim_{\omega\to\infty} F_2(\omega) = \int_0^1 dx\, x\, f(x) \tag{35}$$

$$= \prod_{i=1}^{3} \int_0^1 dx_i\, x_1\, \delta(1-\sum_1^3 x_j)\, f(x_1,x_2,x_3) \tag{36}$$

$$= \frac{1}{3} \prod_{i=1}^{3} \int_0^1 dx_i\, (\Sigma x_j)\, \delta(1-\sum_1^3 x_k)\, f(x_1,x_2,x_3) \tag{37}$$

$$= \frac{1}{3} \prod_{i=1}^{3} \int_0^1 dx_i\, \delta(1-\sum_1^3 x_j)\, f(x_1,x_2,x_3) = 1/3. \tag{38}$$

In a more general n-parton model we would have

$$F_2(\infty) = \frac{1}{n}(\sum_i^n g_i^2). \tag{39}$$

Experimentally, $F_2(w)$ is flat and equals $0.33\pm.04$ for $4 < w < 20$. For w above 20 there may be a fall-off to 0.2 or even to zero, but in this region F_2 depends on the extrapolation of the ratio $R = \sigma_S/\sigma_T$.

For $F_1(w)$ we get from (30)

$$F_1(\infty) = \int_0^1 dx\, f(x) = 1. \tag{40}$$

This is too small, as experimentally $F_1(w)$ seems to be raising above one, possibly going to ∞ with w. This result also contradicts the quite general result by Callan and Gross[7] that

$$F_1(\omega) = \omega\, F_2(\omega).$$

This result was proven assuming only that the electromagnetic current operator has the equal time commutation relations of a product of field operators. This same assumption is the basis of the derivation of the Ward identities. We therefore believe that the discrepancy in F_1 is due to our neglect of Ward identity corrections.

VII. A SIMPLE MODEL FOR THE WEIGHT FUNCTIONS

Scaling merely predicts that both $2mW_1$ and νW_2 approach a function of a single variable $F_{1,2}(x)$; it has nothing to say about the shape of the limiting function. In order to see the quantitative behavior of $F_{1,2}(x)$ it is of interest to have available a simple and perhaps not unreasonable shape for the distribution function $f(x_1, x_2, x_3)$.

The probability that in a fast moving proton any given quark carries a zero fraction of the total momentum is very small. A simple distribution function which gives zero probability for any parton to be at rest is

$$f(x_1, x_2, x_3) = A\, x_1 x_2 x_3 , \tag{42}$$

where A is, a fully determined (33), normalization constant. We have tested this expression and find that it fits the data for νW_2 at best qualitatively. A function which still has all the desirable properties of (42), namely zero probabilities at the end points and leads to integrals which can be done analytically, is

$$f(x_1, x_2, x_3) = A x_1 x_2 x_3 \left(1 + a x_1 x_2 x_3\right). \tag{43}$$

The free parameter a is a measure of how much more peaked (43) is than (42) around the point $x_1 = x_2 = x_3 = 1/3$; at which each quark carries 1/3 of the full momentum.

The normalization condition (33) determines A to be

$$A^{-1} = \int_0^1 dx_1\, dx_2\, dx_3\; x_1 x_2 x_3 \left(1 + a x_1 x_2 x_3\right) \delta\left(1 - \Sigma x_i\right). \tag{44}$$

All the integrals can be done by elementary methods and we find

$$A^{-1} = \frac{1}{120}\left(1 + a/42\right). \tag{45}$$

With this we get for the single particle distribution function

$$f(x) = \int_0^1 dx_2\, dx_3\, \delta(1 - x - x_2 - x_3)\, f(x, x_2, x_3) \tag{46}$$

or

$$f(x) = \frac{28}{a + 42}\, x(1-x)^3 \left[5 + a x (1-x)^2\right]. \tag{47}$$

VIII. THE SCALING FUNCTIONS IN OUR MODEL

Our assumed g(y) (31) can now be inserted into (25) and (30) and leads to

$$F_1(\omega) = \frac{\omega}{\omega-1}\int_{1/\omega}^{1} dx\, x\, f(x) \tag{48}$$

$$F_2(\omega) = \frac{\omega}{\omega-1}\int_{1/\omega}^{1} dx\, f(x). \tag{49}$$

Let z = 1 - 1/w and define the new functions

$$G_{1,2}(z) = F_{1,2}(\omega), \tag{50}$$

then

$$G_2(z) = \frac{1}{z}\int_{1-z}^{1} dx\, x\, f(x). \tag{51}$$

From this we find, using our explicit form (47) for f(u)

$$G_2(z) = \frac{42}{a+42}\left[5h_1(z) + 4ah_2(z)\right], \tag{52}$$

where

$$h_1(z) = z^3 - \frac{8}{5}z^4 + \frac{2}{3}z^5 \tag{53}$$

$$h_2(z) = \frac{1}{6}z^5 - \frac{3}{7}z^6 + \frac{3}{8}z^7 - \frac{1}{9}z^8. \tag{54}$$

Similarly

$$G_1(z) = \frac{1}{z}\int_{1-z}^{1} du\, f(u) \tag{55}$$

or, with (47),

$$G_1(z) = \frac{42}{a+42}\left[5k_1(z) + 4ak_2(z)\right], \tag{56}$$

where

$$k_1(z) = z^3 - \frac{4}{5}z^4 \tag{57}$$

$$k_2(z) = \frac{1}{6}z^5 - \frac{2}{7}z^6 + \frac{1}{8}z^7. \tag{58}$$

In our model both G_1 and G_2 vanish as z^3 at $z = 0$, in agreement with the data. As mentioned before $G_1(1)$ is finite, which is perhaps the biggest failure of our model. The parameter a can be adjusted to give an excellent fit to the data for νW_2. For $a = 130$ this fit is shown in Fig. 4.

IX. APPROACH TO SCALING

The data[4] shows that scaling is reached very fast, the scaling curves are obtained already for $q^2 \sim 1(\mathrm{GeV}/c)^2$. This fact has become known as precoscious scaling. It is therefore of interest to study the approach to scaling for νW_2 in our model.

We go out from (21)

where we now keep the exact kinematical relations

$$2M\nu = t\omega \tag{59}$$

$$s = (p+q)^2 = M^2 + 2p\cdot q + q^2 = M^2 + 2M\nu - t$$
$$= M^2 + t(\omega - 1). \tag{60}$$

This can be used to rewrite the δ-function in (21) as

$$\delta[x^2 M^2 + xt\omega - t - y(M^2 + t(\omega-1))], \tag{61}$$

which implies

$$y = \frac{x^2M^2 + t(x\omega - 1)}{M^2 + t(\omega-1)} \xrightarrow{t\to\infty} \frac{x\omega - 1}{\omega - 1}. \tag{62}$$

The limits of integration on y: $0 \leq y \leq 1$ restrict the lower limit on the x-integral to

$$x_L = \frac{t\omega}{2M^2}\left(-1 + \sqrt{1 + \frac{4M^2}{t\omega^2}}\right) \xrightarrow{t\to\infty} \frac{1}{\omega}. \tag{63}$$

Of course for $t \to \infty$ y and x_L reach the known values which we have indicated. We find the exact expression

$$\nu W_2(t,\omega) = t\omega \int_{x_L}^{1} dx\, f(x)\, x\, g(y(x)) \frac{1}{M^2 + t(\omega-1)}. \tag{64}$$

We now let $g(y) \equiv 1$, to get

$$\nu W_2(t,\omega) = \frac{t\omega}{M^2+t(\omega-1)} \int_{x_L}^{1} dx\, x\, f(x). \tag{65}$$

We can relate this expression easily to the limiting function $G_2(z)$. In fact comparison with (51) shows that

$$\nu W_2(t,\omega) = \frac{t\omega}{M^2+t(\omega-1)} (1-x_L)\, G_2(1-x_L) \tag{66}$$

or

$$\nu W_2(t,\omega) = \frac{\nu}{s} (1-x_L)\, G_2(1-x_L). \tag{67}$$

Noting that $x_L \to 1/w \to 0$, we see that (66) gives us explicitely the approach to scaling. For very low q^2 we can neglect x_L and we see that the scaling limit is reached with a slope w/M^2. This is not instantaneous but still somewhat faster than the data as shown for typical values of w in Figs. 5a and 5b.

X. A SUM RULE

Bjorken and Paschos also showed that we have the sum rule

$$S = \int_0^1 dx\, \nu W_2(x) = \sum_{n=1}^{N} P_n \frac{1}{n}\Big(\sum_1^n Q_i^2\Big). \tag{68}$$

In the ordinary three quark model ($N = 3$, $P_n = \delta_{n3}$) we have for the proton $\Sigma Q_i^2 = 1$ and therefore the above expression has the value 1/3 while experimentally (integration of the area under the curve for F_2) one finds 0.16.

As our model fits the data for νW_2 quite well we expect it also to satisfy the sum rule (68). Substitution of (51) into (68) gives, sucessively

$$S = \int_0^1 d \quad G_2(z) \quad = \int_0^1 \frac{dz}{1-z} \int_z^1 x\, dx\, f(x) \tag{69}$$

$$= \int_0^1 \frac{dz}{z} \int_{1-z}^1 x\, dx\, f(x) . \tag{70}$$

since $G_2(z)$ is given by (51). Using our analytic form (52) for $G_2(z)$ we can do the last integral and find

$$S = A\left[\frac{1}{4} - \frac{8}{25} + \frac{1}{9} + \frac{4a}{5}\left(\frac{1}{36} - \frac{3}{49} + \frac{3}{64} - \frac{1}{81}\right)\right]. \tag{71}$$

A is given in (45), and with a = 130 we find

$$S = 0.19. \tag{72}$$

in good agreement with the experimental value .16.

Our model has shown therefore that due to the mass change of the partons a three quark model can satisfy the above sum rule.

XI. DISCUSSION

We have shown that a variable mass parton can change not only the approach to scaling but also the functions obtained in the scaling limit.

The limiting functions are convolutions of the momentum and mass distribution functions.

We have constructed a simple model using only three quarks to illustrate these points. Our model fits the experimental data on $F_2(w)$, it leads to fast but not instantaneous scaling in agreement with experiment and satisfies the Bjorken-Paschos sum rule.

We have neglected the Ward identity corrections to the parton vertex functions. This does not seem to affect our results on νW_2, but their influence on W_1 should be studied. Our model also predicts for the ratio of the neutron to proton structure functions the constant value 2/3. Experimentally the result is nearly $1 - 1/w$. This is probably due to our neglecting graphs not included in Fig. 3 such as Pomeron exchange. A study of such terms is reserved for a later publication.

APPENDIX I

Here we present a derivation of the discontinuity formula (10) in order to see what input is needed and to obtain the constants. In general we have the time ordered product

$$T_{\mu\nu} = \frac{i}{2}\sum_{s}\int d^4x\, e^{iqx}\langle p,s|\{\theta(x^0)J_\mu(x)J_\nu(0)+\theta(-x^0)J_\nu(0)J_\mu(x)\}|p,s\rangle. \tag{A.1}$$

Note that because of T and P invariance

$$\langle p,s|J_\mu(x)J_\nu(0)|p,s\rangle = \langle p,s|J_\nu(0)J_\mu(-x)|p,s\rangle. \tag{A.2}$$

Therefore

$$T_{\mu\nu}^{*} = \frac{-i}{2}\sum_{s}\int d^4x\, e^{-iqx}\langle p,s|\theta(x^0)J_\nu(0)J_\mu(x)+\theta(-x^0)J_\mu(x)J_\nu(0)|p,s\rangle \tag{A.3}$$

$$= \frac{-i}{2}\sum_{s}\int d^4x\, e^{-iqx}\langle p,s|\theta(x^0)J_\mu(-x)J_\nu(0)+\theta(-x^0)J_\nu(0)J_\mu(-x)|p,s\rangle \tag{A.4}$$

$$= \frac{-i}{2}\sum_{s}\int d^4x\, e^{iqx}\langle p,s|\theta(-x^0)J_\mu(x)J_\nu(0)+\theta(x^0)J_\nu(0)J_\mu(x)|p,s\rangle. \tag{A.5}$$

Thus

$$\mathrm{Im}\, T_{\mu\nu} = \frac{1}{4}\sum_{s}\int d^4x\, e^{iqx}\langle p,s|\{J_\mu(x),J_\nu(0)\}|p,s\rangle. \tag{A.6}$$

For $q^0 > 0$ only one term contributes, thus

$$\mathrm{Im}\, T_{\mu\nu} = \frac{1}{4}\sum_{s}\int d^4x\, e^{iqx}\langle p,s|J_\mu(x)J_\nu(0)|p,s\rangle, \tag{A.7}$$

and, comparing with (A.2)

$$W_{\mu\nu} = 8\pi^2 \frac{E_p}{m} \, \mathrm{Im}\, T_{\mu\nu}. \tag{A.8}$$

This equation expresses the virtual γ - p total cross section in terms of the forward γp- elastic scattering.

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FIGURE CAPTIONS

Figure 1: The process of inelastic γ-p scattering in a three quark model. In this figure the active parton ends up in a meson. There is a similar graph where the active parton enters a baryon (resonance).

Figure 2: The elementary photon-parton process, where the intermediate parton has mass m_2 different from the mass m_1 of the initial parton.

Figure 3: γ-p elastic scattering in a three quark model.

Figure 4: Our best fit to the data in the scaling limit of $\nu W_2(w)$.

Figures 5 (a, b): The approach to scaling. Plotted is $\nu W_2(q^2, w)$ versus q^2 for $w = 5$ in (a) and $w = 20$ in (b). The data are from Ref. 4.

Fig.1

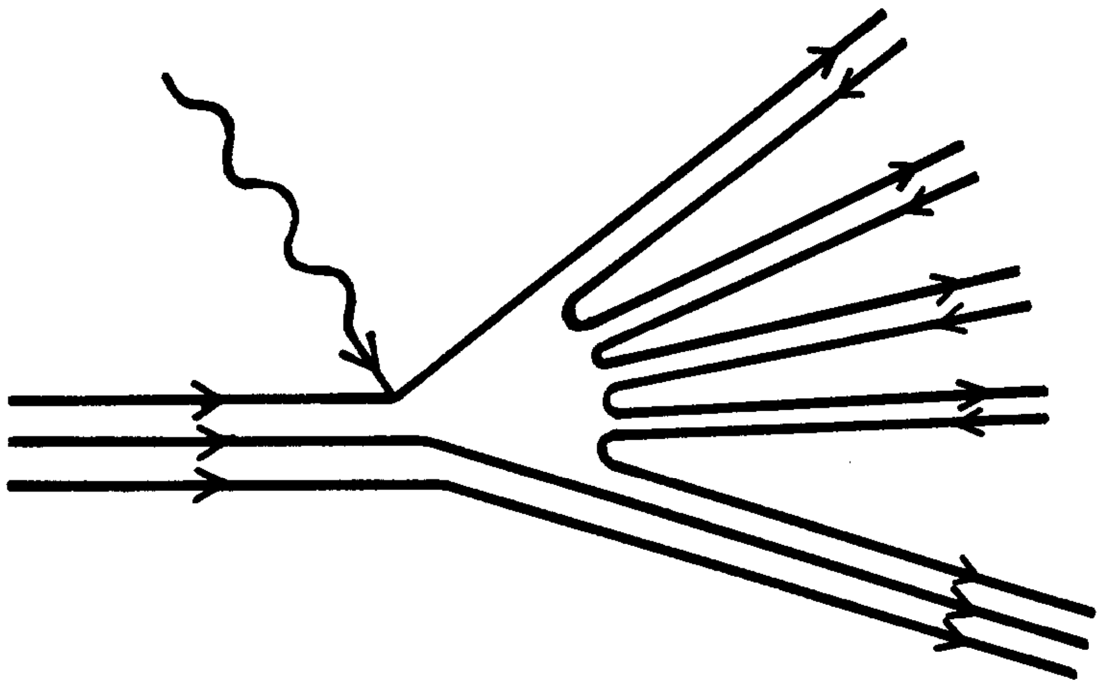


Fig. 2

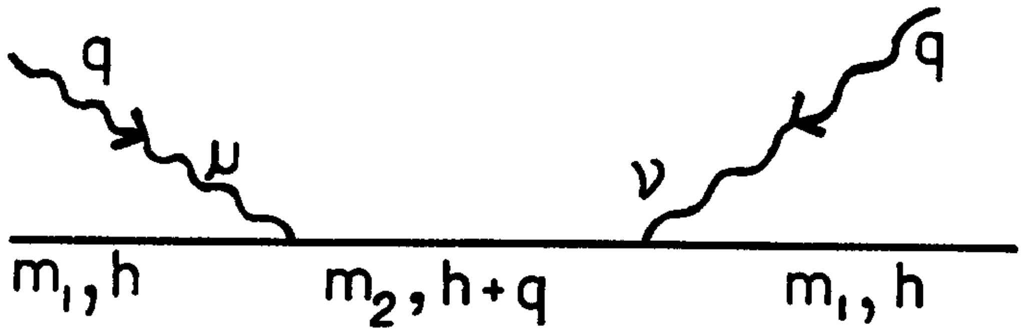


Fig. 3

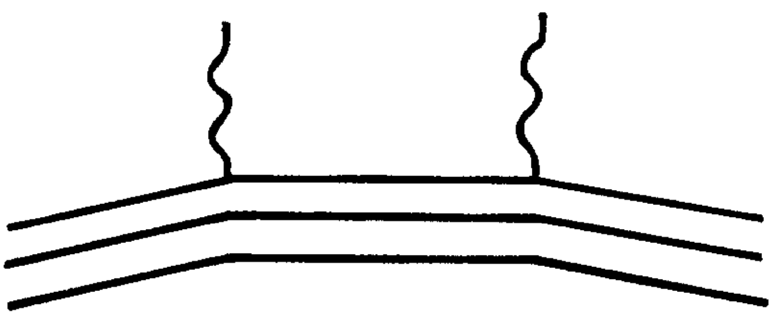
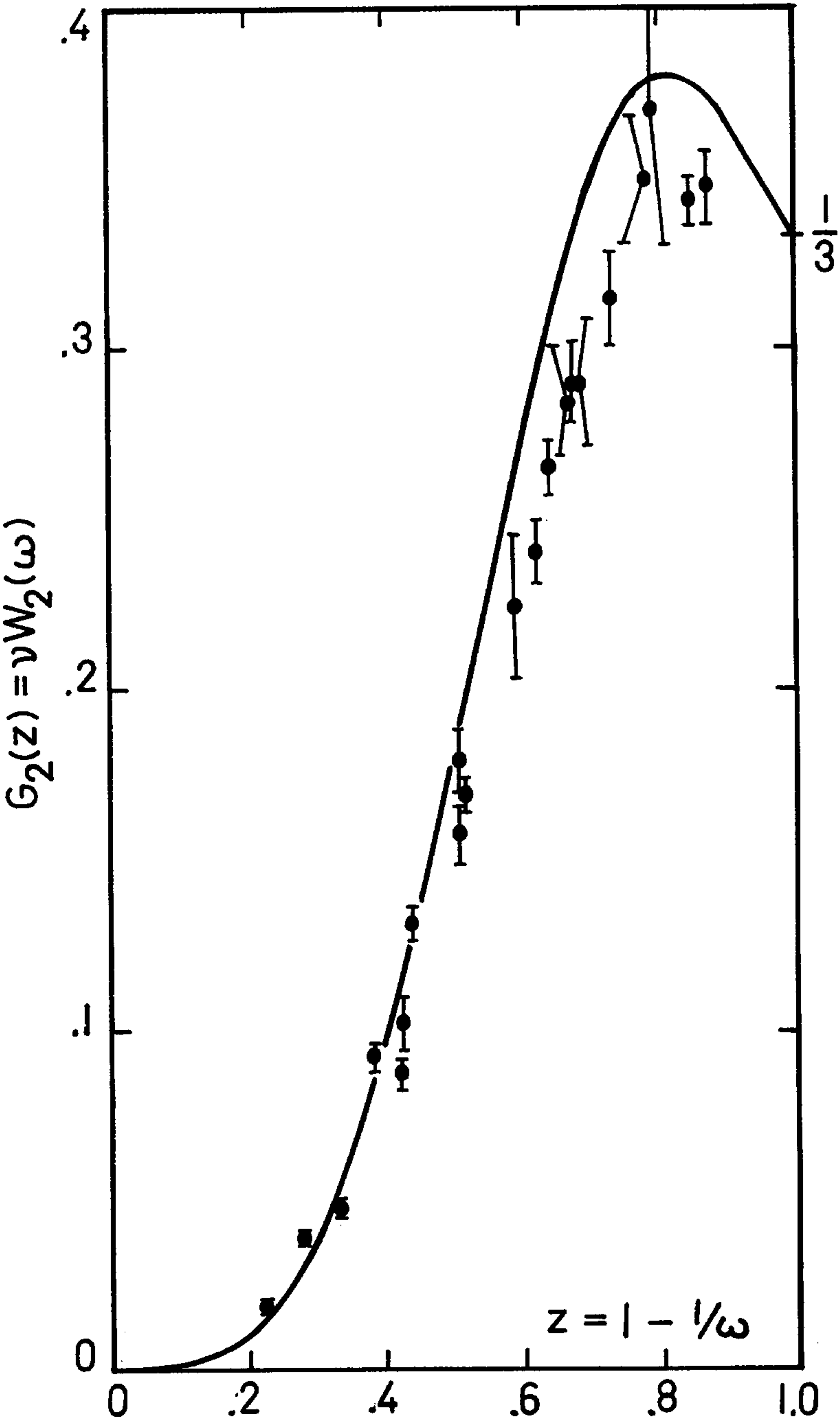


Fig. 4.



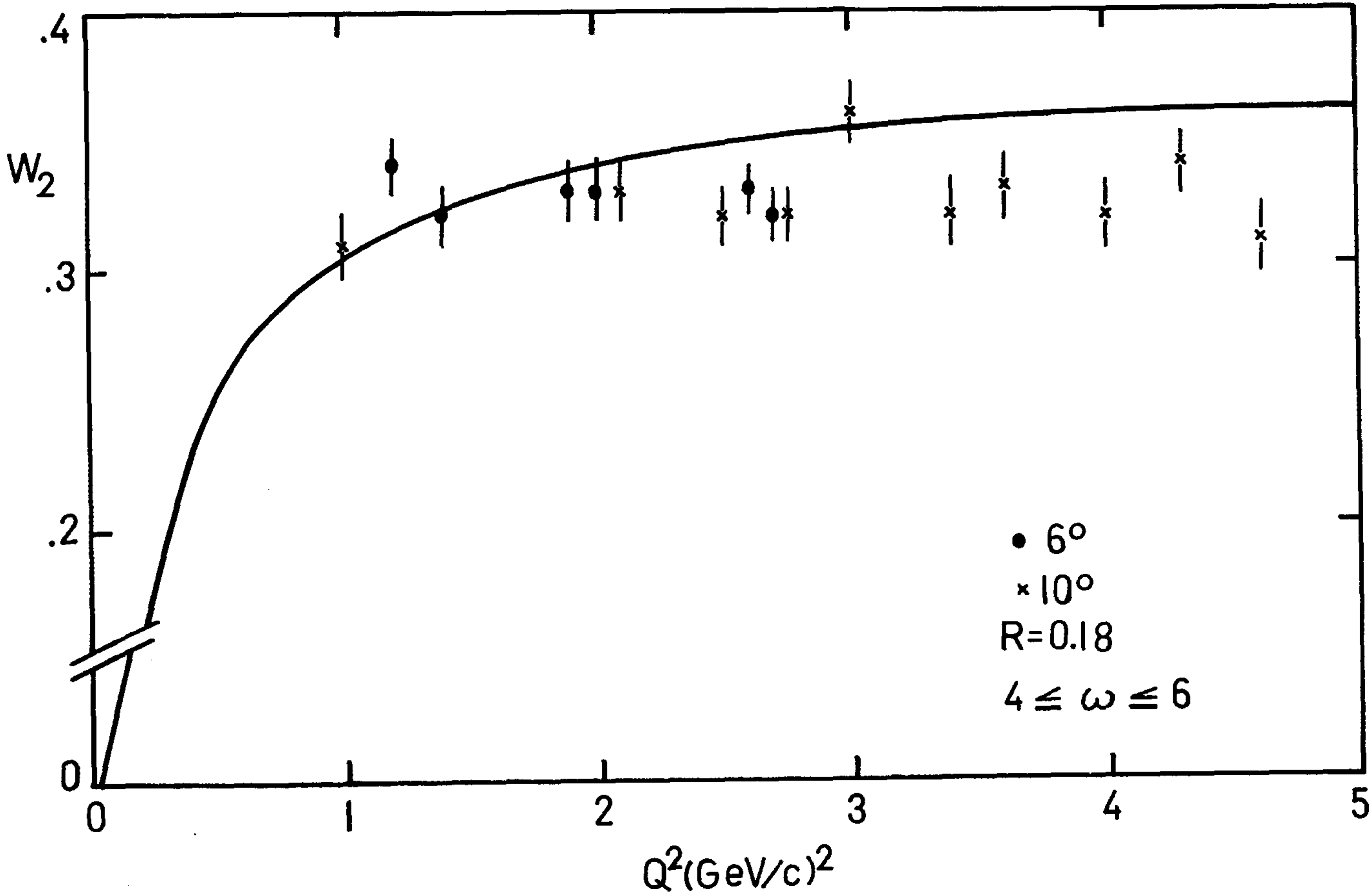
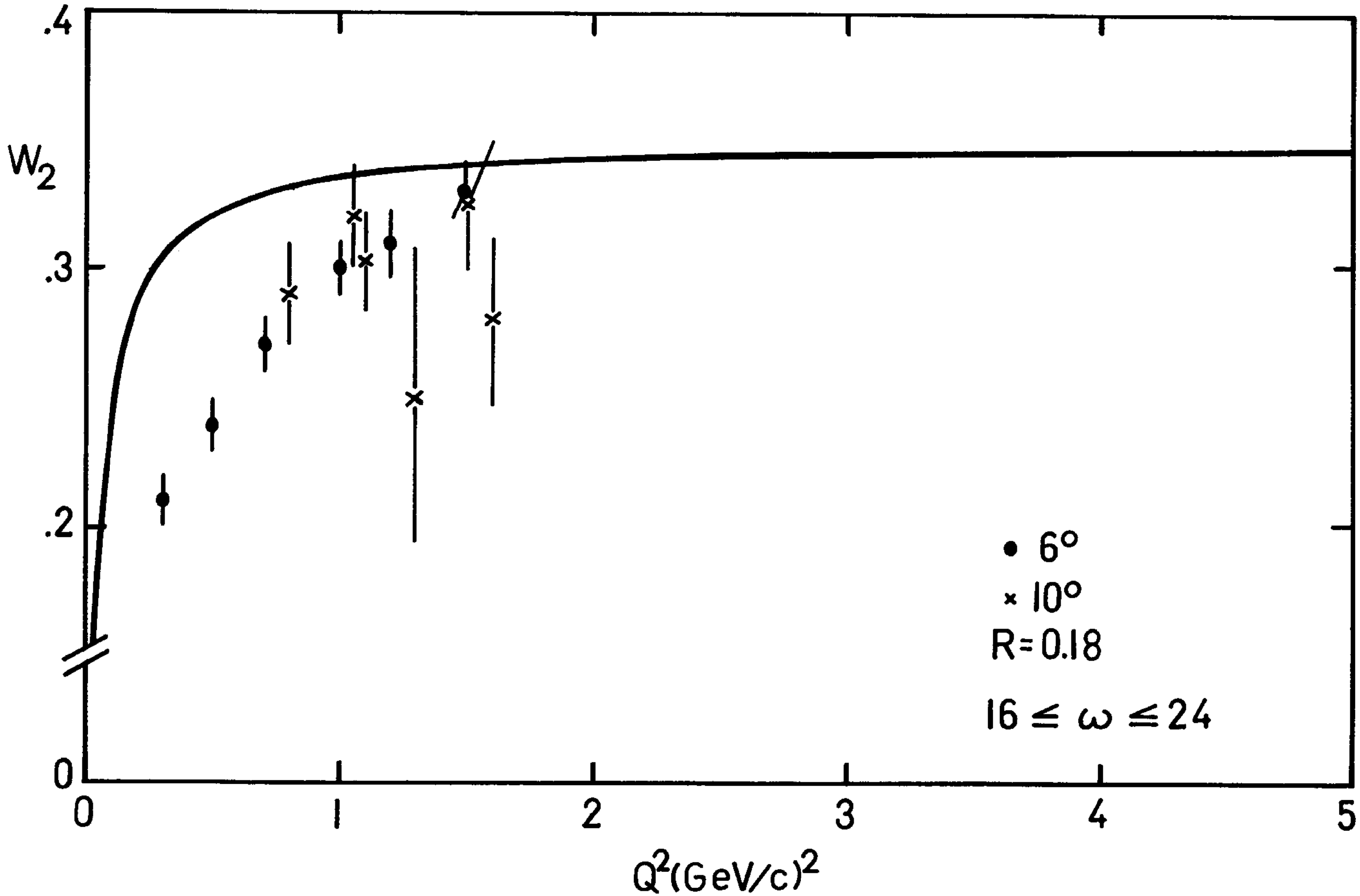


Fig. 5a


W2
.4
.3
.2
0
0
1
2
3
4
5
Q2(GeV/c)2
6°
10°
R=0.18
16 ≤ ω ≤ 24